STAT C-501
STOCHASTIC PROCESSESS AND QUEUING THEORY

STOCHASTIC PROCESSES

1 Let a fair coin is tossed indefinitely. Find the probability that two or more consecutive heads will not occur in n tosses. Also find the generating function of this event.

2 Let $S_N = X_1 + X_2 + ... + X_N$, where N has Poisson distribution with mean a. If X_i's have *i.i.d.* Bernoulli distribution with $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p = q$, show that

(i) S_N has Poisson distribution with mean ap.

(ii) The joint distribution of S_N and N has the probability mass function

$$\Pr(N = n, S_N = y) = \frac{e^{-a} a^n p^y q^{n-y}}{y!(n-y)!} \quad \text{for } n = 0,1,2...; \; y = 0,1,2,...n$$

(i) Cov(N, S_N) = ap.

3 Obtain probability generating function of the random variable X having the mass function

$$p_k = \frac{1}{2} q^{(|k|-1)} (1-q); \quad k = ..., -3, -2, -1, 1, 2, 3, ... \text{ where } 0 < q < 1.$$

(iii)

4 In a series of Bernoulli trials with probability of success p, if X_i denotes the number of failures preceding the i^{th} success and X_{i+1} the number of failures following the i^{th} success but preceding the $(i+1)^{th}$ success $(i = 0,1,2,...,n-1)$, then show that the p.g.f. of is $P_Z(s) = \left(\frac{p}{1-qs}\right)^n$. Hence obtain an expression for $P(Z = r)$.

5 Let $S_N = X_1 + X_2 + ... + X_N$, where X_i's are *i.i.d.* random variables and N is a random variable independent of X_i's. Find expectation and variance of S_N.

6 Let p_n be the probability that n Bernoulli trials result in an even number of successes. Find the generating function of $\{p_n\}$. Hence obtain an expression for p_n. Also find an asymptotic value of p_n using Partial Fraction Theorem.

7 Let X be a non-negative integral valued random variable with probability

$P(X = n) = p_n; \; n = 0,1,2...$, and probability generating function $P(s) = \sum_{n=0}^{\infty} p_n s^n$.

Find the generating function for $P(X > n+1)$.

8 Define convolution of two sequences $\{a_k\}$ and $\{b_j\}$, where $a_k = P(X = k)$ and $b_j = P(Y = j)$ with X and Y being two non-negative integral valued random variables. Find the probability generating function of the sum of two independent random variables

9 Let $X(t) = A_0 + A_1 t + A_2 t^2$, where A_i, i= 0,1,2 are uncorrelated random variables with mean 0 and variance 1. Is $\{X(t), t \in \mathrm{T}\}$ covariance stationary?

10 Let X_n, for n even, takes values $+1$ and -1 each with probability $\frac{1}{2}$, and for n odd, take values $\sqrt{a}$, $\frac{-1}{\sqrt{a}}$, with probabilities $\frac{1}{1+a}$, $\frac{a}{1+a}$ respectively (a is real number > -1 and $\neq 0,1$). Further let X_n's be independent. Is $\{X_n,\ n \geq 1\}$ covariance stationary?

11 Let $Y_n = a_0 X_n + a_1 X_{n-1}$ $(n = 1, 2, 3...)$, where a_0, a_1 are constants and X_n, $n = 1, 2, 3...$ are *i.i.d.* random variables with mean 0 and variance 1. Is $\{Y_n, n \geq 1\}$ covariance stationary?

12 Define the following:

(ii) Closed set
(iii) Ergodic state

13 Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space $S = \{1, 2\}$ with transition matrix

$$P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Find the stationary distribution of given t.p.m.

14 Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space $S = \{1,2,3\}$ and transition matrix

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}.$$

Find the stationary probability distribution of the chain.

15 Suppose that a fair die is tossed. Let the states of X_n be k $(=1,2,3,4,5,6)$, where k is the maximum number shown in the first n tosses. Show that $\{X_n, n \geq 0\}$ is a Markov chain. Find transition probability matrix.

16 Consider a sequence of random variables $X_n,\ n = 0,1,2...$ such that each of X_n assumes only two values -1 and 1 with conditional probabilities

$$P = \begin{pmatrix} 1-a & a \\ b & 1\text{-}b \end{pmatrix};\ 0 < a,b < 1 \ .$$

Let $P(X_0 = 1) = p_1 = 1 - P(X_0 = -1)$ givethe initial distribution. Let

$P(X_n = 1) = p_n;\ q_n = P(X_n = -1) = 1 - p_n$

Then, find p_n, $E(X_n)$ and $corr(X_n, X_{n-1})$.

17 If state k is persistent null, then for every j

$$\lim_{n\to\infty} p^{(n)}{}_{jk} \to 0$$

and if state k is aperiodic, persistent non null then

$$\lim_{n\to\infty} p^{(n)}{}_{jk} \to \frac{F_{jk}}{\mu_{kk}}$$

18 Consider the Markov chain with t.p.m.

$$\begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{2}{5} & 0 & \frac{3}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

Is the chain irreducible? Classify the states of the chain.

19 Six boys Dev (D), Hemant (H), Jeet (J), Mohan (M), Sunil (S) and Tejas (T) play the game of catching a ball. If D has the ball, he is equally likely to throw it to H, M, S and T. If H gets the ball, he is equally likely to throw it to D, J, S, T. If S has the ball, he is equally likely to throw it to D, H, M and T. If either J or T gets the ball, they keep throwing it to each other. If M gets the ball, he runs away with it. Obtain the transition probability matrix and classify the states.

20 Define a persistent state and a transient state. Show that the state j is persistent iff

$$\sum_{n=0}^{\infty} p_{jj}^{n} = \infty$$

21 Consider a Markov Chain $\{X_n, n \geqslant 0\}$ with states 0 and 1 having transition probability matrix

$$X_{n-1} \quad \begin{matrix} & X_n \\ & \begin{matrix} 0 & \quad\quad 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{pmatrix} 1-(1-c)p & (1-c)p \\ (1-c)(1-p) & (1-c)p+c \end{pmatrix} \end{matrix}; \; 0 < p < 1; \; 0 \le c \le 1$$

with initial distribution $P\{X_0 = 1\} = p_1 = 1 - P\{X_0 = 0\}$. Show that correlation coefficient $Corr\{X_{n-k}, X_n\} = c^k$ for $0 < c < 1$.

22 Find the probability of ultimate extinction in the case of the linear growth process ,starting with i individuals at time $t = 0$.

23 Show that $\frac{N(t)}{t}$ provides a consistent estimate for mean rate λ of the Poisson process $\{N(t), t \geq 0\}$.

24 If $\{N(t), t \geq 0\}$ is a Poisson process, then the autocorrelation coefficient between $N(t)$ and $N(t+s)$ is $\sqrt{\frac{t}{t+s}}$.

25 Show that for the linear growth process with immigration having $\lambda_n = n\lambda + a$; $\mu_n = n\mu$. Show that $M(t) = E(X(t))$ satisfies the differential equation $M'(t) = (\lambda - \mu)M(t) + a$. Solve this to get an explicit expression for $M(t)$ with initial condition $M(0) = i$, when $X(0) = i$.

26 State the additive property of a Poisson process. If $\{N_1(t), t \in T\}$ and $\{N_2(t), t \in T\}$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then

(i) Obtain the distribution of $N(t) = N_1(t) + N_2(t)$ and identify the same.

(ii) Find the inter-arrival distribution of $N(t)$.

27 Write down the differential-difference equations for the Yule-Furry process. Obtain an expression for the size distribution $p_n(t) = P(N(t) = n)$, assuming that the process starts with one member at time $t = 0$. Further, obtain the probability generating function of.

28 In the case of $(M/M/1);(\infty/FIFO)$ queuing model, obtain the steady-state probability distribution expressions $\{p_n\}$ of n customers in the system and hence obtain expressions for

(i) Expected number of customers in the system

(ii) The probability that the number of customers is $\geq k$.

(iii) Variance of the queue length.

29 Consider a single server queuing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling units per hour and the expected service time is 0.25 hours .The maximum permissible number of calling units in the system is two.

(i) Derive the steady-state probability distribution of the number of calling units in the system,

(ii) Calculate the probability that system is empty; and

(iii) Find the expected number of calling units in the system.

30 Divide the interval $[0,t]$ into a large number n of small intervals of length h and suppose that in each small interval, Bernoulli trials with probability of success λh and with probability of failure $(1-\lambda h)$ are held. Show that the number of successes in an interval of length t is a Poisson process with mean λt. State the assumptions you make.

31 Show that for the linear growth process, the second moment $M_2(t)$ satisfies the differential equation $M'_2(t) = 2(\lambda - \mu)M_2(t) + (\lambda + \mu)M(t)$. Further, show that variance is

$$Var(X(t)) = i\frac{\lambda+\mu}{\lambda-\mu}e^{(\lambda-\mu)t}\left(e^{(\lambda-\mu)t}-1\right); \lambda \neq \mu$$

where i is the population size at $t = 0$.

32 Define a Poisson process. State the postulates under which a count process will be a Poisson process.

(i) Show that random selection from a Poisson process yields a Poisson process.

(ii) If $N_1(t), N_2(t)$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then obtain the distribution of $N(t) = N_1(t) - N_2(t)$.

33 In the case of $(M/M/1);(N/FCFS)$ queuing model, derive the steady-state probability distribution and, obtain the expressions for

(iv) Expected number of customers in the system

(v) Expected number of customers in the queue

(vi) Expected waiting time in the system

34 A super market has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can

be processed by the cashier is 24 per hour. Calculate

(i) the probability that the cashier is idle

(ii) the average number of customers in the queue

(iii) the average time a customer spends in the system.

35 Describe the classical ruin problem .Derive an expression for the expected duration of the game, which is finite. Obtain the limiting expression as $a \to \infty$

36 Describe the classical ruin problem. Derive an expression for p_z and hence show that $p_z + q_z = 1$, where p_z and q_z are respectively the probabilities of ultimate winning and losing the game for the gambler. Obtain the limiting expression as $a \to \infty$

37 In the classical ruin problem, what will be the effect of reducing the unit at stake from a dollar to half a dollar, on the probability of ruin of the gambler?

38 If X_i's i.i.d. have Logarithmic distribution

$$P(X_i = k) = \frac{\alpha\theta^k}{k}, k = 1, 2, \ldots; \theta > 0, \alpha = \frac{-1}{\log(1-\theta)},$$

and N has a Poisson distribution with mean a , then show that $S_N = X_1 + X_2 + \cdots + X_N$ has negative binomial distribution.

39 Let a_n be the probability that a sequence of n Bernoulli trials result in an even number of successes. Find the generating function $A(s)$ of $\{a_n\}$. Hence show that $a_n = \frac{1}{2}\{1 + (q-p)^n\}$, where p is probability of success in each trial.

40 Consider the Markov chain consisting of states $S = \{1, 2, 3, 4\}$ with the transition probability matrix

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}$$

(i) Classify the states of the above Markov chain

(ii) Find the closed sets, if any and find the stationary distribution of the given chain.

(iii) Find the limiting distribution of P as $n \to \infty$.

41 Consider a three state Markov chain with $S = \{1, 2, 3\}$, initial distribution $\pi_0 = [0.7, 0.2, 0.1]$ and transition probability matrix

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

Find (i) $P(X_1 = 3)$, (ii) $P(X_3 = 1 | X_1 = 2)$ and (iii) $P(X_0 = 1, X_1 = 2, X_2 = 3)$.

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42 In a college 65% of the freshmen advance to the sophomore level, 80% of the sophomores advance to the juniors level, 92% of the juniors advance to the senior level and 95% of the seniors graduate. It is also known that percentages of dropouts (and transfers to other colleges) for each class are: freshmen 25%, sophomores 10%, juniors 3% and seniors 1%. Other students remain at the same level the following year. Determine the transition probability matrix of the Markov chain.

43 Write down the differential-difference equations for the linear growth process with immigration having state dependent transition rates $\lambda_n = n\,\lambda + a$, $\mu_n = n\,\mu$. Show that $M(t) = E[X(t)]$ satisfies the differential equation

$$M'(t) = (\lambda - \mu)M(t) + a.$$

Hence show that with the initial condition $M(0) = i$, when $X(0) = i$,

$$M(t) = \frac{a}{(\lambda - \mu)}\left\{e^{(\lambda-\mu)t} - 1\right\} + i\,e^{(\lambda-\mu)t}; \quad \lambda \neq \mu$$

and taking limit as $\lambda \to \mu$, show that

$$M(t) = a\,t + i;\ \text{for} \quad \lambda = \mu.$$

What is the limit of $M(t)$ as $t \to \infty$ for $\lambda < \mu$?

44 A supermarket has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate

(i) The average number of customers in the queue,
(ii) The average time that a customer spends in the queue,
(iii) The probability that there is no customer waiting to be served.

45 Consider a *counting process* $\{N(t); t \geqslant 0\}$ where $N(t)$ denotes the total number of events that have occurred during an interval of duration t and denote $p_n(t) = Pr\{N(t) = n\}$ for $n = 0, 1, 2, \ldots$. Then by stating the postulates for Poisson process, show that $p_n(t) = \frac{e^{-\lambda\,t}(\lambda\,t)^n}{n!}$, $n = 0, 1, 2, \ldots$.

46 In the classical ruin problem, what is the probability of ruin of the gambler if he is playing against an infinitely rich adversary? Further, obtain an expression for the expected duration of the game, which is finite, when $p = q = \frac{1}{2}$.

47 A petrol station has a single pump and space for not more than 3 cars (2 waiting and 1 being served). A car arriving when the space is filled to capacity goes elsewhere for filling-up of petrol. Cars arrive according to a Poisson distribution at a mean rate of one every 8 minutes. Their services time has an exponential distribution with a mean of 4 minutes.

(i) Find the effective arrival rate.
(ii) What is the probability that an arriving car will get service immediately upon arrival?
(iii) Find the expected number of cars in the system.
(iv) What is the expected waiting time until a car is leaving the petrol pump?

48 Let X be a non-negative integral valued random with probability $P(X = n) = p_n$, $n = 0, 1, 2, \ldots$ and probability generating function $P(s) = \sum_{k=0}^{\infty} p_k s^k$. Obtain the generating functions for the following:

(i) $P(X > n)$, (ii) $P(X = 2n)$.

49 Let X be a random variable denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Find the probability generating function of X. Hence, find the expected number of trials needed.

50 Define convolution. Consider $X_1, X_2, \ldots, X_r$ have i.i.d. Geometric distributions with mean q/p. Show that the convolution of X_i's, $i = 1, 2, \ldots, r$ is Negative Binomial.

51 Define the following:

(iv) Covariance Stationary
(v) Gaussian process
(vi) Markov Chain
(vii) Closed set
(viii) Irreducible Markov Chain
(ix) Ergodic state

52 Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space $S = \{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}.$$

Discuss the nature of the states.

53 Consider a sequence of random variables $X_n, n = 0, 1, 2, \ldots$ such that each of X_n assumes only two values -1 and 1 with conditional probabilities

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0 < a, b < 1.$$

Let $p_n = P(X_n = 1)$, $q_n = P(X_0 = -1) = 1 - p_n$. Further, let $P(X_0 = 1) = p_1 = 1 - P(X_0 = -1)$ be the initial distribution. Then, find $p_n, E[X_n]$, and correlation $corr(X_n, X_{n-1})$.

54 A computer device can be either in a busy mode (state 1) processing a task, or in an idle mode (state 2), when there are no tasks to process. Being in a busy mode, it can finish a task and enter an idle mode any minute with probability 0.2. Being in an idle mode, it receives a new task any minute with probability 0.1 and enters a busy mode. The initial state is idle. Let X_n be the state of the device after n minutes. Find

(i) the distribution of X_1,
(ii) the steady state distribution of X_n,
(iii) the limiting distribution P^n as $n \to \infty$.

55 Write down the differential-difference equations for the Yule-Furry process. Obtain an expression for the size distribution $p_n(t) = Pr[N(t) = n]$, assuming that the process starts with one member at time $t = 0$. Further, obtain the probability generating function of $\{p_n(t)\}$.

56 At a one man barber shop, customers arrive according to Poisson distribution with a mean arrival rate of 5 per hour and his haircutting time was exponentially distributed with an average haircut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate

(i) Average number of customers in the shop and the average number of customers waiting for a haircut.
(ii) The percentage of time an arrival can walk right-in without having to wait.
(iii) The percentage of customers who have to wait prior to getting into the barber's chair.

STAT C-502
STATISTICAL COMPUTING USING C PROGRAMMING

Statistical Computing Using C

1. State whether the following statements are true or false:
   a) The **do-while** statement first executes the loop body and then evaluates the loop control expression.
   b) The **default** case is required in the **switch** statement.
   c) The return type of a function is **int** by default.
   d) Parentheses can be used to change the order of evaluating expressions.
   e) The underscore can be used anywhere in an identifier.
   f) An integer can be added to a pointer.

2. Fill in the blanks.
   a) A program start execution from ____________ function.
   b) The ________ statement when executed in a switch statement causes immediate exit from the structure.
   c) The _________ specification is used to read or write short integer.
   d) The _________ operator returns the number of bytes the operand occupies.
   e) The keyword ___________ can be used to create a data type identifier.
   f) The escape sequence character _________ causes the cursor to move to the next line on the screen.

3. Write the output:

```
f loat x = -4.2,xmin = 4.7;
 if( abs(x) < xmin)  x = (x>0) ? xmin : -xmin ;
 printf("%f", x);
```

4. What is a structure? How does a structure differ from an array?

5. Describe two different ways to access an array element.

6. What is the advantage of data type FILE, describe the different ways in which data type files can be categorized in C.

7. What is a pointer? Why is it necessary to declare the type of pointer? How much memory is allocated to a pointer variable?

8. How do you open/close a data file in C? Why it is important to close the data file? How do you read or write data in these files?

9. What is a period (.) operator? How and where it is used, explain using example.

10. In C, " * " maybe treated as both unary and binary operator. Justify your answer with example?

11. Consider the program segment to answer the following. In this case, assume that the memory addresses of x as 100, y as 300 and u starting from 700

```
double x=20.5, y=10.5, z;
double *px, *py;
int u[3][3] = {{1, 11, 111}, {2, 22, 222}, {3, 33, 333}};
int  *v;
px  =  &x;
py  =  &y;
v   =   &u[1][1];
z    =   (*v + 1)*(*px – y)/2;
```

a) What is the meaning of *px and z ?
b) What is the value of *(v-1) * *(v-4) ?

12. Write a loop that will generate every third integer, beginning with $i = 2$ and counting for all integers that are less than 100. Calculate the sum of those integers that are divisible by 5.

13. Write a conditional expression for the following:
If the variable divisor is not zero, divide the variable dividend by divisor and store the result in variable quotient. If the divisor is zero, assign it to the quotient.

14. Given that $int\ x = 2, y = 3, z = 2, t = -4;$ evaluate the following expressions:
a) $z - (x + z)\%2 + y$
b) $x! = z\&\&\,!\,(y < z)\ ||\ x > t$

15. What are function prototypes in C? What is their purpose? Illustrate with example.

16. Define a self-referential structure contacting the following three members:
a) A 40 element character array called name
b) An integer quantity called lost
c) A floating point quantity called percent

Include the tag team within the structure definition.

17. Find error(s) in the following program:

```
#include<stdio.h>
main();
 {
        int 9x = 2, y;
        scanf("%d", y);
        putchar(\n);
        printf('%c', "A");
        return(0);
 }
```

18. Write the output from the following:

```
#include<stdio.h>
 int a=17;
 main()
 {
   int a=5, b=12, x = 15, y = 2, z = -32765, t = 100;
   float r, s;
   r = x>y ? x/y : x*y;
   s = z + 5;
   b += a;
   a = b – a;
   b = b – a;
   printf("r = %f \n s=%f',r,s);
   printf("a = %d \n b=%d", a, b);
   printf("%d\n", 10 + ++t);
   return (0);
 }
```

19. Write the output from the following:

```
#include<stdio.h>
```

```
int func1(int a);
int func2(int a);
main()
 {
    int a=0, b=1, count;
    for(count = 1; count <= 5; count++)
            {
                        b+=func1(a) + func2(a);
                        printf("%d", b);
            }
}

int func1(int a)
{
    int b;
    b=func2(a);
    return (b);
   }

int func2(int a)
{
    static int b;
    b+=1;
    return (b+a);
   }
```

20. Write the output from the following:

```
#include<stdio.h>
    main()
  {
    int a, b, *p1, *p2, x, y, z;
    a = 12;
    b = 4;
    p1 = &a;
    p2 = &b;
    x  =  *p1  *   *p2  -  6;
    y  =   4*   -   *p2  /   *p1  +  10;
    printf("a = %d, b = %d\n", a, b);
    printf("x = %d, y = %d\n", x, y);
    *p2  =  *p2  +  3;
    *p1  =   *p2  -  5;
    z  =  *p1  *   *p2  -  6;
   printf("a = %d, b = %d\n", a, b);
   printf("z = %d \n", z);
   return (0);
  }
```

21. Answer the questions following the code:

```
void funct(int *p);
main()
{ static int x[ 5]= {1,2,3,4,5};
  funct(x);
}
void funct( int *p)
```

```
{
 int i, prod=0;
 for( i= 0; i< 5; i++)
     prod *= *(p+ i);
  printf( "product= %d", prod);
 return;
}
```

(a) What type of argument is passed to funct?
(b) What value is returned by funct?
(c) What information is passed to funct?
(d) What is the purpose of the for loop in funct?
(e) What is the output of the program?

22. What is a pointer? How can it be initialized? Also, discuss how initial values can be assigned to two dimensional arrays with the help of examples.

23. Describe different forms of loop available in C. How would you decide the use of one of the three loops in C for a given problem?

24. Distinguish between the following with the help of examples:
 a) Global and local variables
 b) Actual and formal arguments

25. Write a C-program to calculate the product of two matrices A and B of order $m \times n$ and $n \times p$ respectively.

26. Write a C-program to fit the Poisson distribution to the following data:

x:	0	1 2	3	4	5	
f:	109	65	22	7	3	1

27. Write a C program to compute the roots of quadratic equation $ax^2 +bx+c=0$.

28. In an experiment on immunization of cattle from tuberculosis, the following results were obtained:

	Affected	Unaffected
Inoculated	12	28
Not inoculated	13	7

Write a C-program to test whether vaccine is effective in controlling the incidence of the disease.

29. Develop a function to draw a random sample of size n from gamma distribution with parameters k and λ. Also find its mean and variance. Hence write a C-program to perform the above mentioned tasks using files.
 Write a C-program to draw a random sample of size n from normal distribution with parameters mu and variance also find its mean and variance.

30. Write a C program to form a frequency table for marks (integers only) (x_i, i = 1, 2, ……n≤ 100) and $0 \leq x_i \leq 50$ with an interval of 10.
31. Given two independent samples (x_i, i=1,2….$n_1 \leq 25$) and (y_i, i=1,2…$n_2 \leq 25$) drawn from the Normal population $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ respectively, Write a C-program to test for the equality of two means using t test.

32. Given the data (x_i, y_i i = 1, 2, ……n$\leq$ 25), develop a C program which first compute ranks and calculate rank correlation between X and Y.

33. Write a C program to fit a Binomial distribution for the given discrete data in the form: { (x_i, f_i) i = 1, 2, 3, ……n $\leq$25}, and also test for goodness of fit.

34. Develop a function to calculate correlation coefficient for the data given on X and Y. Hence, using the function, develop a program to compute multiple correlation coefficient of X on Y and Z.

DSE-1(A)
TIME SERIES

RLA COLLEGE

B.Sc. (H) STATITICS

TIME SERIES

1. Why is the method of least squares not used to fit a modified exponential curve? Describe the method of partial sums to fit a modified exponential curve?

2. Explain how will you decide about the type of trend curve to be fitted to a given time series data, giving appropriate reasons in each case. Discuss the fitting of an exponential curve by the principle of least squares.

3. Describe the moving average method for determining trend in time series. Obtain the trend values for a time series of extent m= 2k+1 if it consist of quadratic polynomial in time variable.

4. Describe the appropriate method for obtaining the seasonal component from a given time series of monthly data when the data is known to have prominent business cycles.

5. Give the genesis of logistic curve and describe Yule's method for fitting this curve.

6. Enumerate the various properties of the curve

   $$U_t = \frac{k}{1+e^{a+bt}} \; ; \; b < 0$$

   Trace the curve and describe its different phases with respect to a time series data of production.

7. Describe the method of "Ratio- to – moving averages" for measuring the seasonal variations stating clearly the assumptions made.

8. It is desired to determine a trend curve in a time series by a weighted moving average method covering a consecutive set of nine points which would accurately represent the series if it consist a cubic polynomial in time variable. Obtain the formula

   $$[9, 3] = \frac{1}{231}[-21, 14, 39, 54, 59, 54, 39, 14, -21]$$

Further, show that the formula is, in fact, good enough, for a quadratic polynomial.

9. What are the different growth curves used for measuring trend? Outline the criteria for selecting a modified exponential curve as trend type. Explain method of three selected points for fitting this curve.

10. Describe Yule's and Hoteling's methods for fitting the logistic curve.

11. What is the meant by seasonal fluctuation of a time series. Describe "Link Relative Method" for measuring the seasonal indices in time series data, stating clearly the assumptions made in this method.

12. Describe the different growth curves? Why is the method of least squares not used to fit a "Grompetrz curve"? Explain method of three selected points for fitting this curve.

13. If $\boldsymbol{\delta}$ stands for the central difference and the series approximated by a cubic show that

$$\frac{1}{hk}[\,h][\,k]\,y_0 = y_0 + \frac{h^2+k^2-2}{24}\,\delta^2 y_0$$

Where $\frac{1}{h}\,[h]$ stands for simple average of 'h' terms. Hence deduce Spencer's 21 point formulae. Verify the weights are symmetric about the middle value and sum of weight is unity.

14. Why can't we measure the random component of a time series? Describe the variate difference method to estimate the variance of the random component of a time series with its significance.

15. Name different components of a time series. Obtain a suitable weighted moving average formula for measuring trend covering consecutive sets of seven points which would accurately represent the series if it consist of a quadratic polynomial in a time variable.

16. Explain autocorrelation function. for infinite series generated by moving average of a random component with equal weights, the correlogram is given by :

$$r_k = \begin{cases} 1-\frac{k}{m}, & for\ k \le m \\ 0, & for\ k > m \end{cases}$$

Where k is the order of the serial correlation and m is the length of the moving average system.

17. For the auto regressive series of the type

$$y_{t+2} + \alpha y_{t+1} + \beta y_t = \mathcal{E}_{t+2}$$

Show that the complete solution for large series is

$$y_t = \sum_{j=0}^{\infty} \xi_j \mathcal{E}_{t-j+1}$$

18. Describe the Auto regressive process of order one and two.

19. Explain weak stationarity. Estimate the parameter of AR (1) and AR (2) with "Yule-Walker equation".

20. Where we can use "Exponential smoothing method". Describe "Exponential smoothing method" for future forecasting.

21. Explain the following
    - Brown's discounted regression
    - Box-Jenkins method
    - Bayesian forecasting
    - Moving average process

DSE-2(A)
OPERATIONAL RESEARCH

Ram Lal Anand College
Statistics(H)
Operational Research

1. A small jewellery manufacturing company employs a person who is highly skilled gem cutter, and it wishes to use this person at least 6 hours per day for this purpose. On the other hand, the polishing facilities can be used in any amount up to 8 hours per day. The company specializes in three kinds of semiprecious stones P, Q and R. Relevant cutting, Polishing and cost requirements are listed in the following table. How many gemstones of each type should be processed each day to minimize the cost of finished stones? What is the minimum cost?

	P	Q	R
Cutting	2 hr	1 hr	1 hr
Polishing	1 hr	1 hr	2 hr
Cost per stone	Rs. 30	Rs. 30	Rs. 10

2. The following data are available for a firm which manufactures three items A, B and C:

Product	Time required(in hrs.)		Profit
	Assembly	Finishing	
A	10	2	800
B	4	5	600
C	5	4	300
Firm's capacity	2,000	1,009	

(i) Formulate the l.p.p. ,

(ii) Solve the formulated l.p.p. so as to maximize the profit from production

3. Food A contains 20 units of Vitamin X and 40 units of vitamin Y per gram. Food B contains 30 units each of Vitamin X and vitamin Y per gram. The daily minimum human requirements of vitamin X and vitamin Y are 900 units and 1200 units respectively. Use the principle of duality to determine that how many grams of each type of the food should be consumed so as to minimize the cost if food A costs 60 paisa per gram and food B costs 80 paisa per gram.
4. An electronics firm is undecided as to the most profitable mix for its products. The products manufactured are transistors, resistors and carbon tubes with a profit (per 100 units) of Rs. 10, Rs. 6 and Rs. 4 respectively. To produce a shipment of transistors containing 100 units requires I hour of engineering, 10 hours of direct labour and 2 hours of administrative service. To produce 100 resistors requires 1 hour, 4 hours and 2 hours of engineering, direct labour and administrative service respectively. To produce one shipment of tubes (100 tubes) requires 1 hour of engineering, 5 hours of direct labour and 6 hours of administrative service. There are 100 hour of engineering, 600 hours of direct labour and 300 hours of administration.

   (i) Express the above data in the form of L.P.P

   (ii) Solve using the simplex method to find the most profitable mix and maximum profit.

5. Use the principle of duality to solve the following L.P.P.

Minimize $z = 10\,y_1 + 8\,y_2$

subject to the constraints:

$$4y_1 + 2y_2 \geq 5$$
$$2y_1 + 2y_2 \geq 3$$
$$y_1,\ y_2 \geq 0.$$

6. Consider the following LP problem

$$\text{Max } z = 3x_1 + 2x_2 + 5x_3$$

subject to the constraints:

$$x_1 + 2x_2 + x_3 \leq a_1$$
$$3x_1 + 2x_3 \leq a_2$$
$$x_1 + 4x_2 \leq a_3$$

where a_1, a_2, a_3 are constants. For specific values of a_1, a_2, a_3 the optimal solution is

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
Z	4	0	0	c_1	c_2	0	1350
x_2	b_1	1	0	½	-1/4	0	100
x_3	b_2	0	1	0	½	0	c_3
x_6	b_3	0	0	-2	1	1	20

where b_i 's and c_i 's are constants.

Determine:

(i) The values of a_1, a_2 and a_3 that yield the given optimal solution.

(ii) The values of b_1, b_2 and b_3 & c_1, c_2, c_3 in the optimal tableau.

(iii) The optimal dual solution.

7. A firm manufacturing office furniture provided the following information regarding resource consumption, availability and profit contribution:

Resources	Usage per unit			Availability
	Tables	Chairs	Book case	
Timber (cu. ft.)	8	4	3	640
Assembly deptt.(man hours)	4	6	2	540
Finishing deptt.(man hours)	1	1	1	100
Profit contribution per unit(in □)	30	20	12	

(i) Formulate the problem as a linear programming problem.

(ii) Find the optimal product mix and the total maximum profit contribution.

8. Consider the following L.P.P.:

$$Max. Z = 4x_1 + 10x_2$$

subject to constraints:

$$2x_1 + x_2 \leq a_1$$
$$2x_1 + 5x_2 \leq a_2$$
$$2x_1 + 3x_2 \leq a_3$$
$$x_1, x_2 \geq 0$$

where a_1, a_2 *and* a_3 are constants. For specific values of a_1, a_2 *and* a_3 the optimal solution is:

Basis	x_B	x_1	x_2	x_3	x_4	x_5
x_3	30	b_1	0	1	$\frac{-1}{5}$	0
x_2	20	b_2	1	0	$\frac{1}{5}$	0
x_5	30	b_3	0	0	$\frac{-3}{5}$	1
$Z_j - c_j$	Z =200	0	0	0	d	e

Evaluate the following:

(i) The values of a_1, a_2 *and* a_3 that yield the given optimal solution.

(ii) The values of b_1, b_2 *and* b_3, d *and* e in the above given optimal table.

9. Consider the following linear programming problem:

$Max. Z = 60x_1 + 80x_2$

subject to constraints:

$$6x_1 + 5x_2 \leq 900$$
$$3x_1 + 5x_2 \leq 600$$
$$x_1, x_2 \geq 0$$

The optimal solution to the above problem is given below:

Basis	x_B	x_1	x_2	x_3	x_4
x_1	100	1	0	$\frac{1}{3}$	$\frac{-1}{3}$
x_2	60	0	1	$\frac{-1}{5}$	$\frac{2}{5}$
$Z_j - c_j$	Z =10,800	0	0	-4	-12

If in the above linear programming problem another activity x_k with coefficient 65 and constraint utilization coefficients 3 and 3 are introduced then develop the new optimal solution using the concept of post optimality.

10. Discuss the effect of adding a new non-negative variable x_k in the given l.p.p.:

Max. $z= 3x_1+ 4x_2+ x_3 +7x_4$

subject to the constraints:

$8x_1+ 3x_2+ 4x_3+ x_4 \leq 7$

$2x_1+6x_2+x_3+5x_4 \leq 3$

$x_1+4x_2+5x_3+2x_4 \leq 8$

$x_j \geq 0$; j=1, 2, 3 ,4 .

It is given that the coefficient of x_k in the constraints are 2,7 and 3 respectively and the cost component associated with x_k is 5.

11. Solve the given the l.p.p. ;

$$\text{Minimize } z = 3x_1 + 6x_2 + x_3$$

subject to the constraints:

$x_1 + x_2 + x_3 \geq 6$

$x_1 + 5x_2 - x_3 \geq 4$

$x_1 + 5x_2 + x_3 \geq 24$

$x_1, x_2, x_3 \geq 0.$

Discuss the effect of changing the requirement vector [6, 4, 24] to [6, 2, 12]on the optimum solution.

12. Use dual simplex method to solve the following problem:

Min $Z = 2x_1 + x_2 + 3x_3$

subject to the constraints:

$x_1 - 2x_2 + x_3 \geq 4$

$2x_1 + x_2 + x_3 \leq 8$

$x_1 - x_3 \geq 0$

$x_1, x_2, x_3 \geq 0$.

13. Use dual simplex method to solve the following l.p.p. :

Max $Z = -3x_1 - 2x_2$

subject to the constaints:

$x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_1 \leq 3$ and $x_1, x_2 \geq 0.$

14. Use the principle of duality to solve the following l.p.p. :

$$Min. Z = 2x_1 + x_2 + 3x_3$$

subject to constraints:

$$2x_1 + 3x_2 + 4x_3 \geq 20$$

$$4x_1 + 2x_2 + 2x_3 \geq 15$$

$$x_1, x_2, x_3 \geq 0$$

15. Use dual simplex method to solve the following problem:

$$\text{Min } Z = 2x_1 + x_2 + 3x_3$$

subject to the constraints:

$$x_1 - 2x_2 + x_3 \geq 4$$

$$2x_1 + x_2 + x_3 \leq 8$$

$$x_1 - x_3 \geq 0$$

$$x_1, x_2, x_3 \geq 0$$

16. Use dual simplex method to solve the following problem:

$$\text{Min } Z = 10x_1 + 6x_2 + 2x_3$$

subject to the constraints:

$$-x_1 + x_2 + x_3 \geq 1$$

$$3x_1 + x_2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0 .$$

17. XYZ company buys in lots of 500 boxes which is a 3 month supply. The cost per box is □ 125 and the ordering cost is □150. The inventory carrying cost is estimated at 20% of unit value. What is the total cost of the existing inventory policy? How much money could be saved by employing economic order quantity?

18. An aircraft uses rivets at an approximately constant rate of 5,000 kgs. per year. The rivets cost □ 20 per kg. and the company purchase manager estimate that it costs □200 to place an order, and the carrying cost of inventory is 10% per year. How frequently should orders for rivets be placed and what quantities should be ordered for?

19. A Company has a demand of 12000 units/year for an item and it can produce 2000 such items per month. The cost of one setup is Rs. 400 and the holding cost/unit/month is Rs 0.15. Find the optimum lot size and the total cost per year, assuming the cost/unit is Rs. 4. Also find the maximum inventory, manufacturing time and total time.

20. A manufacturer has to supply 12,000 units of a product per year to his customer. The demand is fixed and known and the shortage cost is assumed to be infinite. The inventory holding cost is Rs. 0.20 per unit per month and the set up cost per run is Rs. 350. Determine the economic order quantity and the minimum total variable yearly cost.

21. Obtain an expression for the economic order quantity for an inventory model with infinite role of replenishment when shortages are allowed.

22. Obtain an expression for the economic order quantity for an inventory model with finite rate of replenishment when shortages are not allowed.

23. Obtain the expression for economic order quantity for an inventory model with uniform demand and several production runs of unequal length and the production is instantaneous.

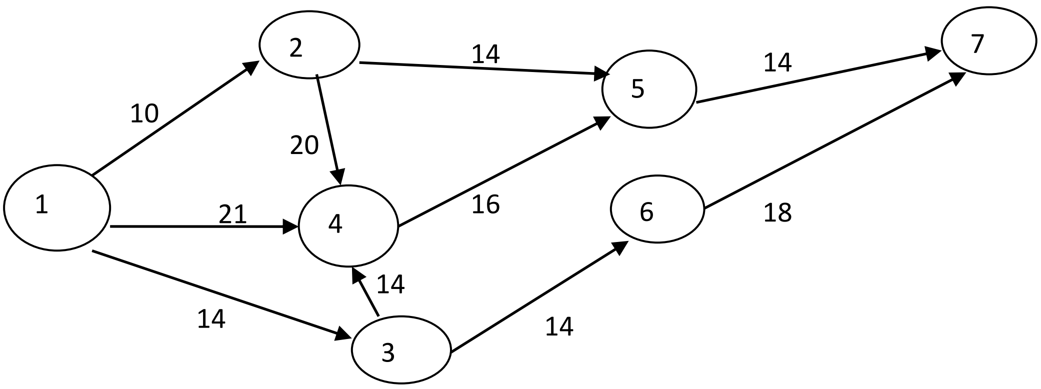
24. Obtain an expression for the economic order quantity for an inventory model with infinite rate of replenishment when shortages are not allowed.

25. ABC company is engaged in manufacturing five brands of packed snacks; B_1, B_2, B_3, B4, B5. It is having five manufacturing set-ups; S_1, S_2, S_3, S_4, S_5, each capable of manufacturing any of its brands one at a time. The cost to make a brand (in □)on these set-ups vary according to the following table:

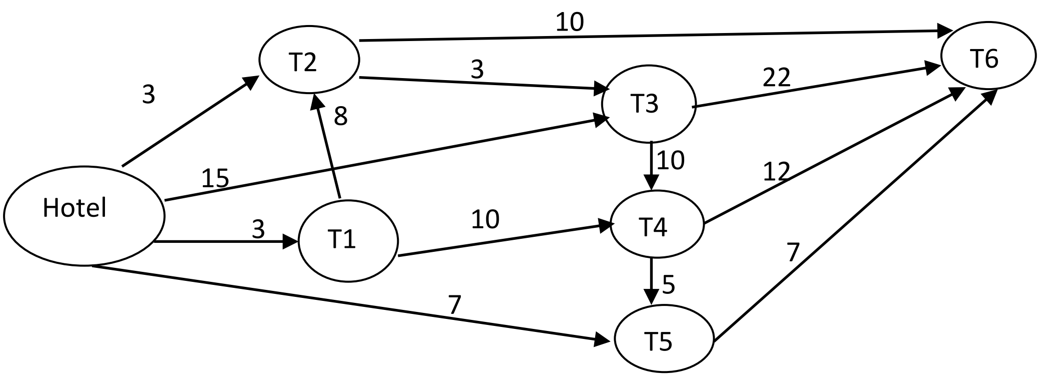
Set-ups / Brand	S_1	S_2	S_3	S_4	S_5
B_1	4	6	7	5	11
B_2	7	3	6	9	5
B_3	8	5	4	6	9
B_4	9	12	7	11	10
B_5	7	5	9	8	11

Obtain the optimal assignment of products on these set-ups resulting in minimum cost.

26. A hotel manager had to drive daily between his residence and distant place of work. Due to heavy traffic on roads during the peak hours it became difficult for him to drive. He decided to approach a radio taxi company but before doing that he decided to determine the shortest route to the work place from his residence. The following net work gives the permissible routes and their distances in kms. between his residence (node 1) and six places (node 2 to node 7) . Determine the shortest route and the shortest distance from his residence (node 1) to the work place (node 7):



27. In a travel guide map a distance (in miles) from the hotel to all the places of tourist interest (T1 to T6) is given below in network flow diagram. Obtain the shortest route and shortest distance to each tourist place from the hotel for the tourist.



28. A bank manager had to drive daily between his residence and distant place of work. Due to heavy traffic on roads during the peak hours it became difficult for him to drive. He decided to approach a radio taxi company but before doing that he decided to determine the shortest route to the work place from his residence. The following table gives the permissible routes and their distances in km. between his residence (node 1) and six places (node 2 to node 7) . Determine the shortest route and the shortest distance from his residence (node 1) to the work place (node 7).

Nodes	1	2	3	4	5	6	7
1		11	15	20			
2				22	14		
3				15		13	
4					18	16	
5							15
6							18

29. The network below gives the permissible routes and their lengths in miles between city 1 (node 1) and six other cities (node 2-7). Determine the shortest route and hence the shortest distance from city 1 to node 7 (city 7).

Nodes	1	2	3	4	5	6	7
1		12	16	19			
2				20	17		
3				13		14	
4					17	18	
5							16
6							20

30. Five salesmen are to be assigned to five territories. Based on the past performance, the following table shows the quarterly sales (in million rupees) that can be generated by each salesman in each territory. Find the optimal assignment.

Salesman	Territory				
	T1	T2	T3	T4	T5
S1	3	8	2	10	3
S2	8	7	2	9	7
S3	6	4	2	7	5
S4	8	4	2	3	5
S5	9	10	6	9	10

31. A garment manufacturer plans to add four regional warehouses to meet the increased demand. Solve the problem for optimal assignment when the following bids (in lacs of Rupees) have been given for the construction of the warehouses:

	Warehouse				
		A	B	C	D
Contractor	1	30	27	31	39
	2	28	18	28	37
	3	33	17	29	41
	4	27	18	30	43
	5	40	20	27	36

32. A company is faced with a problem of assigning 4 machines to 6 different jobs (one machine to one job only).The profits are estimated as follows:

Job	*Machines*			
	A	B	C	D
1	3	6	2	6
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
5	5	2	4	3
6	5	7	6	4

Solve the problem to maximize the total profits.

33. Two players A and B plays a game, each has coins of denominations Rs. 1, Rs. 2, Rs. 5 and Rs. 10. Each selects a coin without the knowledge of other's choice. If the sum of the coins is an odd amount, A wins B's coin, if the sum is even, B wins A's coin. Find the best strategy for each of the player and comment on value of the game.

34. Solve the following game :

Player A	Player B					
	I	II	III	IV	V	VI
1	4	2	0	2	1	1
2	4	3	1	3	2	2
3	4	3	7	-5	1	2
4	4	3	4	-1	2	2
5	4	3	3	-2	2	2

35. Solve the following game:

	Player B				
		B1	B2	B3	B4
PlayerA	A1	19	6	7	5
	A2	7	3	14	6
	A3	12	8	18	4
	A4	8	7	13	-1

36. Solve the following game whose pay-off matrix is given by:

Player B						
		B_1	B_2	B_3	B_4	B_5
Player A	A_1	9	3	1	8	0
	A_2	6	5	4	6	7
	A_3	2	4	3	3	8
	A_4	5	6	2	2	1

37. A state has four government hospitals A, B, C and D. Their monthly requirements of medicines are met by four distribution centres X, Y, Z and W. The data in respect of a particular item vis-à-vis availabilities at the centres, requirements at the hospitals and the distribution cost per unit (in paise) are given below:

Warehouse	Hospital				Availability
	A	B	C	D	
X	44	84	84	80	2000
Y	92	30	64	80	12000
Z	32	100	96	72	5000
W	80	36	120	60	6000
Requirement	8000	8000	6000	3000	25000

Determine the optimum distribution.

38. Find the optimum solution to the following transportation problem:

	Warehouse				
Factory	**D**	**E**	**F**	**G**	**Capacity**
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Demand	80	90	110	160	

39. XYZ firm has three factories located throughout a country. The daily oil production at each factory is as follows:

Factory 1; **18 million** litres, Factory 2; 3 **million** litres, Factory 3; **21 million** litres.

Each day the firm must fulfil the needs of its four distribution centres. Minimum requirement at each centre is as follows:

Distribution centre 1; **21 million** litres, Distribution centre 2 ; **15 million** litres, Distribution centre 3 ; **9 million** litres, Distribution centre 4 ; **6 million** litres.

Cost of shipping one million litres (in □**)** from each plant to each distribution centre is given in the following table in thousands of rupees:

Factory	Distribution Centre			
	D_1	D_2	D_3	D_4
F1	8	12	44	28
F2	4	0	24	4
F3	20	32	60	36

Obtain the optimal distribution for XYZ firm to minimize the shipping costs.

40. A transport company ships truckloads of grains from three warehouses with supply capacity of 25, 25 and 10 truckloads to four mills with the demand of 5, 15, 15 and 15 truckloads respectively. The unit transportation cost per truckload on the different routes is given below:

Mills

Warehouse	1	2	3	4
A	10	2	20	11
B	12	7	9	20
C	4	14	16	18

Solve the problem to get the optimal transportation schedule in order to minimize the transportation cost.

41. Write short notes on the following:

(i) Assignment problem as a special case of Transportation problem
(ii) Two person zero sum game.
(iii) Canonical and standard form of linear programming
(iv) Post optimal analysis – changes affecting feasibility and optimality
(v) Primal dual relationship and optimal primal solution from dual
(vi) Hungarian method for solving assignment model.
(vii) Transportation problem as a linear programming model
(viii) Dominance and modified dominance in a competitive game
(ix) General linear programming problem
(x) Pure and mixed strategy in a game
(xi) Duality in a linear programming problem
(xii) Unbalanced transportation problem

DSE-2(B)
ECONOMETRICS

Econometrics Question Bank

1. Consider a linear parametric function $c'\beta$ where c is a k x 1 column vector of known constants. Let $c'\hat{\beta}$ be an unbiased estimator of $c'\beta$. Prove that $c'\hat{\beta}$ is the best linear unbiased estimator of $c'\beta$ among the class of all linear unbiased estimators of $c'\beta$.

2. For the generalized least squares model $Y = X\beta + u$, where β satisfies linear restrictions $R\beta = r$ where r is a known column vector and R is a known matrix, obtain the unbiased estimate *b* of β and variance covariance matrix of *b*.
3. For the linear model $Y = X\beta + u$, derive the test to test the hypothesis H_0: $\beta_i = 0$, that is X_i has no linear influence on Y.
4. For the model $y = \alpha + \beta x + u$, obtain the predicted value of y and its 95% confidence limits for $x = x_o$.
5. Define a general linear model, state all the assumptions. Show that e = Mu where $M = (I_n - X(X'X)^{-1}X')$ and hence obtain an unbiased estimator of σ_u^2.
6. For the following general linear model : $Y_t = 275.2 + 382.2\ X_{2t} + 185.3\ X_{3t} + U_t$; $R^2 = 0.8114$ and $F = 16.48$; (a) Obtain the sample size *n* (b) Interpret the values of the partial regression coefficients.
7. Explain the concept of co-efficient of determination R^2 and adjusted R^2. Derive the relationship between R^2 and adjusted R^2.
8. Given the following regression:

$$\hat{Y}_t = 263.64 - .0056\ X_{2t} - 2.2316\ X_{3t}$$

Se =	(11.59)	(0.0019)	(0.2099)	
t =	(22.74)	(-2.82)	(-10.63)	R^2 =0.7077
p-value =	(0.0000)	(0.0065)	(0.0000)	

   Where Y is Child Mortality, X_2 is per capita GNP, X_3 is the female literacy rate. Interpret the partial regression coefficients and the intercept and comment on their statistical significance. Also comment on the value of R^2.
9. What do you understand by multicllinearity? Consider a simple model $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + U_t$; Show for the given model that the precision of the estimation falls i.e. (i) specific estimates may have large errors, (ii) these errors may be highly correlated and (iii) sampling variance of the coefficients will be large.
10. Explain all the steps of Farrar Glauber test to test the presence of multicollinearity in the given data.
11. Discuss in details four ways of addressing the problem of multicollinearity.
12. Discuss Frisch's Confluence analysis for detecting the presence of multicollinearity.

13. For the generalized least square model $Y = X\beta + u$, with $E(uu') = \sigma^2\Omega$, σ^2 being unknown and Ω being known, symmetric positive definite matrix, Compute Aitken's estimator for β, its variance and σ^2.

14. For the generalized least squares model $Y = X\beta + u$ with $E(u) = 0$ and $E(uu') = V$, where V is assumed to be known, symmetric positive definite matrix, find the best linear unbiased predictor of a single value of the regressand y_0, given the row vector of prediction regressors x_0.
15. For the generalized least squares model $Y = X\beta + u$ with $E(uu') = \sigma^2 \Omega$, σ^2 being unknown and Ω being known, symmetric positive definite matrix, and

$$1. \quad \Omega = \begin{bmatrix} \frac{1}{\lambda_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\lambda_2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\lambda_n} \end{bmatrix}$$

λ's being known positive numbers. Further, suppose that the variance of the disturbance term is proportional to 1/X i.e. $E(u_i^2) = \sigma^2 / X_i$. Find out the variance covariance matrices of β_{ols} and $b_{Aitkens}$ for the case of single explanatory variable.
16. Describe the Glejser test for dealing with the heteroscedastic disturbances. Also discuss the powers and limitations of Glejser test.
17. Describe the Goldfield and Quandt test in details.
18. Briefly outline Cocharan Orcutt iterative procedure and Durbin's two stage procedure for estimating the parameters of the model $Y = \alpha + \beta X + u$.
19. Define auto-correlation. Give reasons for the presence of auto correlation. Given the model $y = \alpha + \beta x + u$. Assume the first order autoregressive scheme $u_t = \rho\, u_{t-1} + \varepsilon_t$ where ε_t satisfies the assumptions of the classical linear regression model. (a) Show that $Var(u_t) = \sigma^2/(1 - \rho^2)$, where σ^2 is the variance of ε_t. (b) What is the covariance between u_t and u_{t-1} ? Between u_t and u_{t-2}? Generalize your results. (c) Write the variance-covariance matrix of u's. What happens if ordinary least square estimation is applied to the model suffering from auto-correlated disturbances?
20. What do you mean by lag? Discuss Koyck's geometric lag scheme of reducing the number of parameters to be estimated.